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FREE TRANSVERSE VIBRATION ANALYSIS OF UNIFORM SHORT BEAMS

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ABSTRACT
In this work, analytical models like Timoshenko Beam Theory (TBT) and Euler-Bernoulli Beam Theory (EBT) are used for doing the free vibration analysis of a short cantilever beam for estimating the natural frequencies for the first four principal modes of an arbitrary length cantilever beam which satisfies the short cantilever beam criterion and also for estimating the fundamental natural frequencies for the variation of the length of the short cantilever beam. The cross-section of the beam is rectangular and the material used is Mild Steel. The obtained results from both the theories are compared with the simulation results using ANSYS R18.2 for validation. The results obtained from Timoshenko beam model are very nearer to those of simulation results using ANSYS R18.2 than those obtained from the Euler-Bernoulli beam model. Hence, Timoshenko beam model being the accurate one in analyzing the short beams for their free vibration analysis.

Keywords: *Short rectangular cantilever beam, Mild steel, Free vibration analysis, Timoshenko Beam Theory, and Euler-Bernoulli Beam Theory, ANSYS R18.2*

I. INTRODUCTION

All real structures behave dynamically when subjected to loads or displacements. There are two most popular theories for analyzing the beams which behave dynamically when subjected to loads or displacements, one is Euler-Bernoulli beam theory and the other one is Timoshenko beam theory. The primary effects of beams subjected to load are transverse deflection due to pure bending and transverse inertia and the secondary effects are shear deformation and rotatory inertia of the cross-section of the beam. The governing equation which includes the primary effects only was derived from Euler-Bernoulli beam theory and that which include secondary effects along with primary effects was derived by Timoshenko [3]. The Euler-Bernoulli beam theory can give the natural frequencies of flexural vibrations of lower modes of longcantilever beams (L/D>7) quite accurately. For higher modes of longcantilever beams and for short cantilever beams (L/D<7) Timoshenko beam theory will give the accurate values for their dynamic analysis.

The effects of shear deformation and rotatory inertia were first introduced in the vibrating beam equations derived by Timoshenko, S. P. in the year 1921. Timoshenko, in that paper, considered a value of 2/3 as shear coefficient, *K* for rectangular cross section [1].

The shear coefficient, *K* is introduced to allow for the fact that the shear stress is not uniform over the cross section. According to the commonly accepted definition, K is the ratio of the average shear strain on a section to the shear strain at the centroid. The coefficient *K*is a dimensionless quantity, dependent on the shape of the cross section, which is considered because the shear stress and shear strain are not uniformly distributed over the cross section. The one-dimensional theory of beams can be improved by considering the transverse shear deformations and, in the case of vibrating beams, rotary inertia. The beam equations which consider these effects are generally called as Timoshenko's beam equations [1, 2] and they have received considerable attention in the literature. In these equations the effective transverse shear strain is calculated by dividing the product of the shear modulus and the shear coefficient, *K.*

A lot of research has been done on choosing a correct value for the shear coefficient, K. Timoshenko also used the values $K = (6+12\vartheta+6v^2) / (7+12\vartheta+4v^2)$ for the circular and $K = (5+5\vartheta)/(6+5\vartheta)$ for the rectangular cross sections and are closest to the experimental values [2].

Cowper, G. R., 1966, derived the formulae for the shear coefficients for various like circular, hollow circular, rectangular, elliptical, semi-circular, and thin-walled round tubular, square tubular, I-Section, Box section, Spar-And-Web, T-Section cross sections for the case of static problems while deriving the equations of Timoshenko's beam theory by integration of the equations of three-dimensional elasticity theory. The numerical values obtained from above formulae are agree with Timoshenko's values when Poisson's ratio value, ϑ is taken as zero [4].

Kaneko concluded that the values for K obtained from Timoshenko's [2] equations are closest to experimental values [5].

Hutchinson and Zilmer compared their three dimensional series solution and a plane stress solution for the completely free beam with the Timoshenko beam theory for rectangular cross section. The plane stress solution is in good agreement with Timoshenko beam theory using Timoshenko's shear coefficient, K [6].

However, in this work, the value obtained from the equation, $K = (5+5\vartheta)/(6+5\vartheta)$ [2] that was suggested by Timoshenko, S. P. for shear coefficient for rectangular cross section is used.

II. DIFFERENTIAL EQUATIONS FOR FREE TRANSVERSE VIBRATIONS OF A BEAM

Here for free transverse vibrational analysis a cantilever beam is considered.

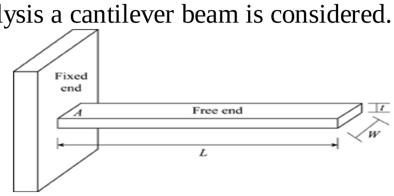

Fig 1. Cantilever Beam

A. Differential equations for free transverse vibrations of a uniform timoshenko beam [8]

In Timoshenko beam theory the effects of shear deformation (SD) and rotary inertia (RI) are considered for the flexural vibrations of a uniform short beam. The loading condition and free body diagram of a cantilever beam according to Timoshenko Beam Theory is shown in the below Fig 2.

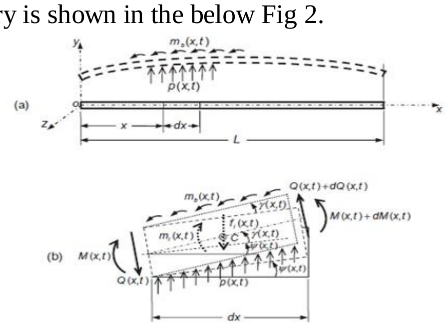

Fig 2. A Transversely Vibrating Timoshenko beam

a) External loads, p(x) and m_b(x, t), in the coordinate system oxyz;

b) Free - body diagram for the beam segment dx.

For free transverse vibration analysis of a beam, we can neglect external force per unit length, ***p(x, t)*** and external bending moment per unit length, $m_b(x, t)$.

For the free transverse vibration of a uniform Timoshenko beam, the coupled equations for the total deflection, *y*(*x*, *t*) and the rotation due to bending moment, $\psi(x,t)$ are given by

$$\rho A\frac{\partial^2 y(x,t)}{\partial t^2} - KGA\left[\frac{\partial^2 y(x,t)}{\partial x^2} - \frac{\partial\psi(x,t)}{\partial x}\right] = 0 \qquad (1a)$$

$$EIz\frac{\partial^2\psi(x,t)}{\partial x^2} + KGA\left[\frac{\partial y(x,t)}{\partial x} - \psi(x,t)\right] - \rho Iz\frac{\partial^2\psi(x,t)}{\partial t^2} = 0 \qquad (1b)$$

Where, *E*, *Iz*, ρ, *A*, *K*, *G* aremodulus of elasticity, second moment of area, mass density, cross-sectional area, shear coefficient and shear modulus of the beam respectively. And also here *K*, the shearcoefficient is considered to account for the variation of shear strain across the cross-section of the beam.
Eliminating $\psi(x,t)$ or *y(x, t)* from the equations (1a) and (1b) we respectively get two differential equations in *y(x, t)* and $\psi(x,t)$ as follows:

$$EIz\frac{\partial^4 y(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} - \rho Iz\left(1+\frac{E}{KG}\right)\frac{\partial^4 y(x,t)}{\partial x^2\partial t^2} + \frac{\rho^2 Iz\partial^4 y(x,t)}{KG\partial t^4} = 0 \qquad (2a)$$

$$EIz\frac{\partial^4\psi(x,t)}{\partial x^4} + \rho A\frac{\partial^2\psi(x,t)}{\partial t^2} - \rho Iz\left(1+\frac{E}{KG}\right)\frac{\partial^4\psi(x,t)}{\partial x^2\partial t^2} + \frac{\rho^2 Iz\partial^4\psi(x,t)}{KG\partial t^4} = 0 \qquad (2b)$$

The slope of the Timoshenko beam is given by the following equation:

$$\frac{\partial y(x,t)}{\partial x} = \psi(x,t) + \gamma(x,t) \qquad (3a)$$

Where,
$\psi(x,t)$ Is the rotational angle due to the bending moment and $\gamma(x,t)$ is the shear strain due to the shearing force. From the above equation one can has

$$\gamma(x,t) = \frac{\partial y(x,t)}{\partial x} - \psi(x,t) \qquad (3b)$$

For the case of free vibrations, the translational and rotational displacement functions respectively will be as follows:

$$y(x,t) = Y(x)e^{i\omega t} \quad (4a)$$

$$\psi(x,t) = \boldsymbol{\psi}(x)e^{i\omega t} \qquad (4b)$$

Where,
Y(x) and $\boldsymbol{\psi}$ (*x*) are the amplitudes of *y(x, t)* and $\psi(x,t)$ respectively, and ω is the natural frequency in rad/sec, t is the time in seconds and i=$\sqrt{-1}$.

Now, the solutions for the equations 2(a) and 2(b) after substituting the equations (4a) and (4b) and eliminating the common term $e^{i\omega t}$ will be as follows:

$$Y(x) = \bar{A}cosh\delta x + \bar{B}sinh\delta x + \bar{C}cos\varepsilon x + \bar{D}sin\varepsilon x \qquad (5a)$$

Where, the constants $\bar{A}, \bar{B}, \bar{C}, and\ \bar{D}$ are for the translational displacement function *Y(x)*, and

$$\boldsymbol{\psi}(x) = A'sin\delta x + B'cos\delta x + C'sin\varepsilon x + D'cos\varepsilon x \qquad (5b)$$

Where, the constants *A'*, *B'*, *C'*, and *D'* are for the rotational displacement function $\boldsymbol{\psi}$(*x*).

The equations *Y(x)*, $\boldsymbol{\psi}$(*x*) represent the translational and rotational mode shapes of the uniform Timoshenko beam respectively.
Herein, equations (5a, b),

$$\delta = \sqrt{\frac{1}{2}\left(-\alpha + \sqrt{\alpha^2 + 4\beta^4}\right)}$$

$$\varepsilon = \sqrt{\frac{1}{2}\left(\alpha + \sqrt{\alpha^2 + 4\beta^4}\right)} \qquad \text{(6a, b)}$$

Where,

$$\alpha = \frac{\rho Iz\left(1 + \frac{E}{KG}\right)\omega^2}{EIz}$$

$$\beta^4 = \frac{\rho A\omega^2 - \frac{\rho^2 Iz}{KG}\omega^4}{EIz} \text{(7a, b)}$$

$A' = a\bar{B}$, $B' = a\bar{A}$, $C' = -b\bar{D}$ and $D' = -b\bar{C}$(8a, b)
Where,

$$a = \frac{KGA\delta}{(-EIz\delta^2 - \rho Iz\omega^2 + KGA)},$$

$$b = \frac{KGA\varepsilon}{(-EIz\varepsilon^2 - \rho Iz\omega^2 + KGA)} \text{(9a, b)}$$

Also, on substituting equations (4a, b) and $\gamma(x,t) = \Gamma(x)e^{i\omega t}$ into equations (3b) and then inserting equations (5a, b) into the resulting expression, one obtains the shear deformations displacement function as follows:

$$\Gamma(x) = Y'(x) - \psi(x) = cA\sinh\delta x + cB\cosh\delta x - dC\sin\varepsilon x + dD\cos\varepsilon x \qquad (10)$$

Where,
$c = \delta - a, \qquad d = \varepsilon - b$(11a, b)
We can determine the integrating constants $\bar{A}, \bar{B}, \bar{C}, and\ \bar{D}$ by using boundary conditions of the beam.
For free vibrations, we have

$$Q(x,t) = Q(x)e^{i\omega t},$$
$$M(x,t) = M(x)e^{i\omega t}, and$$
$$\gamma(x,t) = \Gamma(x)e^{i\omega t} \text{ (12a, b, c)}$$

Where,
Q(x), *M(x)*, and Γ(*x*) represent the amplitudes of shear force, *Q(x, t)*, bending moment, *M(x, t)*, and shear strain, *γ(x, t)* respectively. And also we have

$$Q(x) = KGA\,\Gamma(x) \qquad \text{(13a)}$$
$$M(x) = EIz\,\boldsymbol{\psi}'(x) \qquad \text{(13b)}$$

B. Boundary conditions for calculatingintegrating constants$\bar{A}, \bar{B}, \bar{C}$, and $\bar{D}$:

For Free End:

$$M(x) = EIz\,\boldsymbol{\psi}'(x) = 0 \text{ and } Q(x) = KGA\,\Gamma(x) = 0 \qquad (14a, b)$$

For Clamped End:

$$Y(x) = 0 \text{ and } \boldsymbol{\psi}(x) = 0 \qquad (15a, b)$$

For Hinged End:

$Y(x) = 0$ and $M(x) = EIz\,\boldsymbol{\psi}'(x) = 0$ (16a, b)

From the equations (15a, b) and (14a, b) the boundary conditions for C-F beam are respectively as follows:

$$Y(0) = 0, \boldsymbol{\psi}(0) = 0 \qquad (17a, b)$$
$$\boldsymbol{\psi}'(L) = 0, \Gamma(L) = 0 \qquad (18a, b)$$

From the equations (5a), (5b) and (10), and (17a, b), (18a, b), we have

$$Y(0) = \bar{A} + \bar{C} = 0 \qquad \text{(19a)}$$
$$\boldsymbol{\psi}(0) = a\bar{B} + b\bar{D} = 0 \qquad (19b)$$
$$\boldsymbol{\psi}'(L) = a\delta\bar{A}\cosh\delta L + a\delta\bar{B}\sinh\delta L - b\varepsilon\bar{C}\cos\varepsilon L - b\varepsilon\bar{D}\sin\varepsilon L = 0 \text{ (19c)}$$
$$\Gamma(L) = c\bar{A}\sinh\delta L + c\bar{B}\cosh\delta L - d\bar{C}\sin\varepsilon L + d\bar{D}\cos\varepsilon L = 0 \text{ (19d)}$$

For non-trivial solution of equations (19a)-(19d) requires that

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$$\Delta(\omega) = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & a & 0 & b \\ a\delta cosh\,\delta L & a\delta\ sinh\ \delta L & -b\varepsilon\ cos\varepsilon L & -b\varepsilon\ sin\ \varepsilon L \\ c\ \sinh \delta L & c\ \cosh \delta L & -d\ \sin \varepsilon L & d\ \cos \varepsilon L \end{vmatrix} = 0 \quad (20)$$

and is the frequency equation of the C-F uniform Timoshenko beam which is a transcend equation.

Now, from equations (6a, b), (7a, b), (9a, b), and (11a, b), the equation (20) is a function of ω, and the any one of the numerical methods (such as half-interval method) can be used to calculate the natural frequencies ω_r (r=1, 2, 3 … represents mode number) [10].

C. Differential equations for free transverse vibration of a uniform euler-bernoulli beam [8]

For a uniform free transverse vibration of slender beam EBT neglect the external force per unit length *p(x, t)*, and external bending moment per unit length, $m_b(x, t)$.

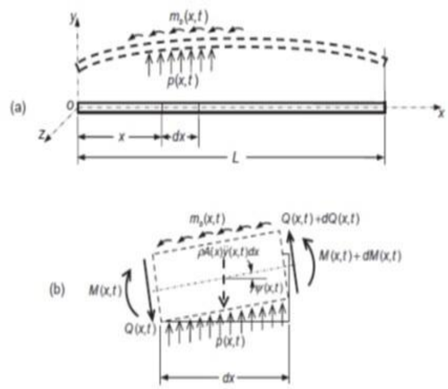

Fig 2. A Transversely Vibrating Euler-Bernoulli beam

- ***a) The external force, p(x) and the external moment, $m_b(x, t)$, in the coordinate system oxyz;***
- ***b) The free-body diagram for the differential beam segment dx.***

According to Euler-Bernoulli beam theory which neglects the shear deformation (SD) and rotary inertia (RI), the differential equations for free transverse vibration are:

$$Q(x,t) = -EIz\frac{\partial^3 y(x,t)}{\partial x^3} \quad (21)$$

$$EIz\frac{\partial^4 y(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} = 0 \quad (22)$$

Where, *Q(x, t)* is the shear force and *E, Iz,ρ, and A* are the young's modulus, second moment of the area of the beam cross section, mass density, and cross-sectional area of the uniform beam respectively. Consider the displacement function as:

$$y(x,t) = Y\,(x)e^{i\omega t} \quad (23)$$

Where, $Y(x)$ is the amplitude of *y(x, t)*, ω is the natural frequency of the beam, t is the time, and i=$\sqrt{-1}$.
From equations (22) and (23), we will get

$$Y''''(x) - \beta^4 Y(x) = 0 \quad (24)$$

or

$$\omega = (\beta L)^2\sqrt{\frac{EI_Z}{(\rho A L^4)}} \quad (25)$$

Now, the solution of the equation (24) will be in the form:

$$Y(x) = B_1 e^{\beta x} + B_2 e^{-\beta x} + B_3 e^{i\beta x} + B_4 e^{-i\beta x} \qquad (26)$$

Where,
$B_1, B_2, B_3, and\ B_4$ are constants of integration.
And since,

$$e^{\pm\beta x} = cosh\beta x \pm sinh\beta x, e^{\pm i\beta x} = cosh\beta x \pm i\ sinh\beta \qquad (27)$$

Equation (26) can also be expressed as:

$$Y(x) = C_1 cosh\beta x + C_2 sinh\beta x + C_3 cos\beta x + C_4 sin\beta x \qquad (28)$$

Which is the natural mode shape of the uniform Euler-Bernoulli beam with the integration constants C_1-C_4 which can be determined from boundary conditions of the beam.

D. Boundary conditions for calculating integrating constants c_1, c_2, c_3, c_4

Once again recalling

$$\psi(x,t) = \frac{\partial y(x,t)}{\partial x} \qquad (29)$$

is the bending slope,

$$\boldsymbol{\psi}(\boldsymbol{x}) = Y'(x) \qquad (30)$$

is the amplitude of $\psi(x,t)$,

$$\bar{M}(x,t) = EIzY''(x) \qquad (31)$$

is the amplitude of bending moment, and

$$\bar{Q}(x) = -EIzY'''(x) \qquad (32)$$

is the amplitude of *Q(x, t),*
Now, boundary conditions for determining the integrating constants C_1-C_4 are similar to as expressed in equations (14a, b), 15(a, b), and (16a, b) for free, clamped, and hinged ends respectively and are as follows:
At the clamped end (*x*=0):

$$Y(0) = 0, \ Y'(0) = 0 \qquad (33a, b)$$

At the free end (*x*=L):

$$Y''(L) = 0, Y'''(L) = 0 \qquad (34a, b)$$

Now, from the equations (28), (33a, b) we will get

$$C3 = -C1, C4 = -C2 \qquad (35a, b)$$

On substituting the equations (35a, b) into equation (28) we will get

$$Y = C1\ (cosh\beta x - cos\beta x) + C2(sinh\beta x - sin\beta x) \qquad (36)$$

Now from the equations (36a) and (34a, b), we will obtain

$$C1\ (cosh\beta L + cos\beta L) + C2(sinh\beta L + sin\beta L) = 0 \qquad (37a)$$

$$C1\ (sinh\beta L - sin\beta L) + C2\ (cosh\beta L + cos\beta L) = 0 \qquad (37b)$$

For the non-trivial solution of the simultaneous equations (37a, b), it requires that

$$\Delta(\omega) = \begin{vmatrix} \cosh\beta L + \cos\beta L & \sinh\beta L + \sin\beta L \\ sinh\beta L - sin\beta L & cosh\beta L + cos\beta L \end{vmatrix} = 0 \qquad (38)$$

or

$$\cosh\beta L \cos\beta L = -1 (39)$$

or

$$\cos\beta L = -\frac{1}{\cosh\beta L} \qquad (40)$$

The above equation (40) is called the frequency of the Euler-Bernoulli beam and for the solution of this equation (40) we can use a numerical method like Half- Interval method [10].

III. RESULTS AND DISCUSIONS

For validating the fundamental natural frequencies obtained from EBT and TBT we considered a numerical example of a cantilever beam with the variation of lengths as 137.5 mm, 112.5 mm, 87.5 mm, and 62.5 mm and the cross-sectional dimensions as 24.75 mm width and 10 mm thickness. The material used is mild steel. The element selected for the beam definition is BEAM 2D 188 in ANSYS R18.2 [9].

Table 1. Mechanical Properties of Mild Steel

S. No.	Mechanical Property	Value
1	Young's Modulus	$1.96 \times \mathbf{10^{11}}$Pa
2	Density	7850 Kg/$\boldsymbol{m^3}$
3	Poison's Ratio	0.3

Ansys R18.2 results[9]:

The following figures named as Fig 4, 5, 6, and 7 are the fundamental mode shapes and the corresponding fundamental natural frequencies of mild steel cantilever beams of various effective lengths 137.5 mm, 112.5 mm, 87.5 mm, and 62.5 mm respectively obtained in ANSYS R18.2.

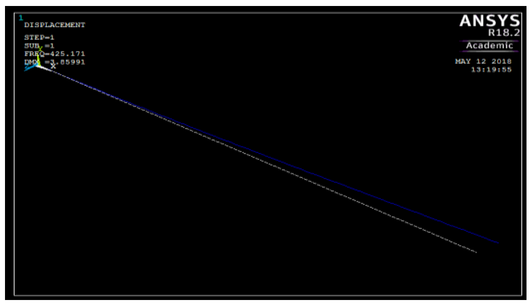

Fig 4. For Length, L=137.5 mm

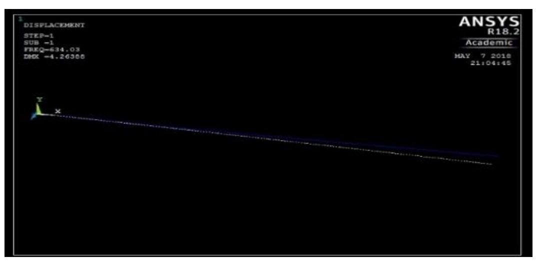

Fig 5. For Length, L=112.5 mm

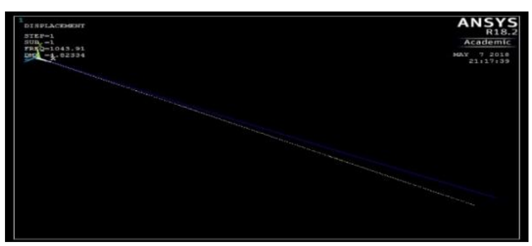

Fig 6. For Length, L=87.5

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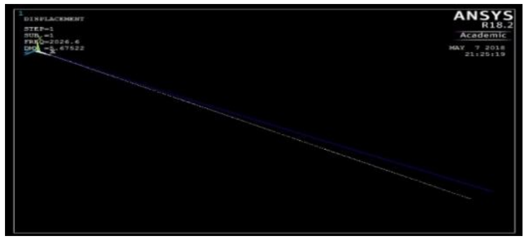
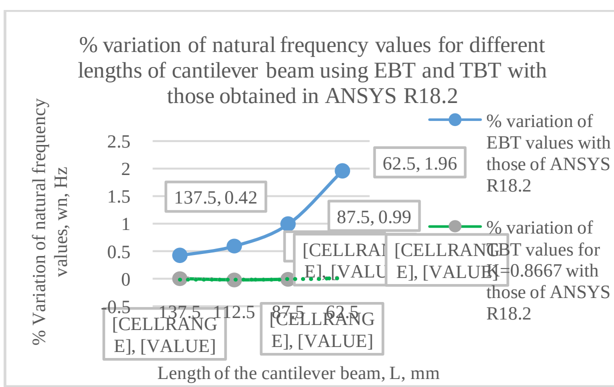


Fig 7. For Length, L=62.5 mm

Table 2. comparison of ansys r18.2 results with the results of ebt and tbt for different lengths of mild steel cantilever beam

S. No.	ANSYS R18.2 Results	EBT Results	TBT Results	% variation of EBT values with those of ANSYS R18.2	% variation of TBT values for K=0.8667 with those of ANSYS R18.2
			K=0.8667		
L, mm	ω_n, Hz	ω_n, Hz	ω_n, Hz		
137.5	425.17	426.95	425.18	0.42	0.00235
112.5	634.03	637.78	633.9	0.59	-0.0205
87.5	1043.91	1054.3	1043.8	0.99	-0.0105
62.5	2026.6	2066.4	2026.8	1.96	0.0099

From Table 2, we can observe that the fundamental natural frequency values of a cantilever beam will be increased as the length of the cantilever beam is decreased. We can also observe that the fundamental natural frequency values obtained for identical cantilever beams in length and cross-section (rectangular) from Timoshenko Beam Theory (TBT) are nearer to the ANSYS R18.2 values than those obtained from Euler-Bernoulli Theory (EBT). The percentage variation of the fundamental natural frequency value for the decrease in length of the cantilever beam for their identical cross-section (rectangular) obtained from the Euler-Bernoulli Theory (EBT) is increased more than that from Timoshenko Beam Theory (TBT) when compared with the values obtained from ANSYS R18.2



Graph 1. Percentage variation of fundamental natural frequency values for different lengths of cantilever beam using EBT and TBT with those obtained in ANSYS R18.2.

The following figures named as Fig 8, 9, 10, 11 respectively show the first four principal mode shapes and the corresponding natural frequencies of the mild steel cantilever beam of length, L=137.5 mm in ANSYS R18.2.

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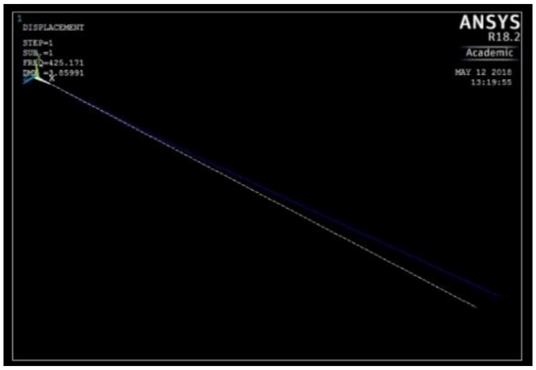

Fig 8. Principal Mode 1

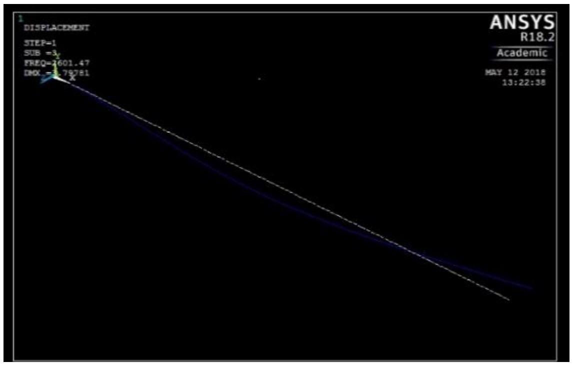

Fig. 9 Principal Mode 2

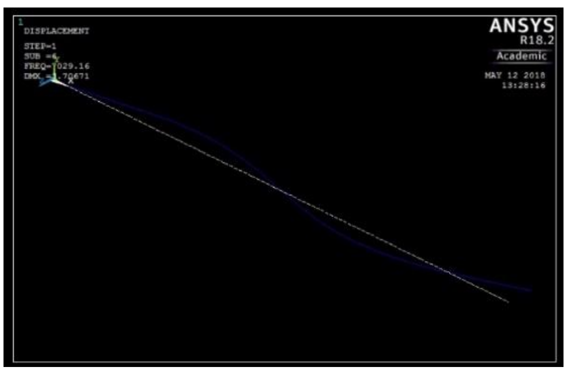

Fig. 10 Principal Mode 3

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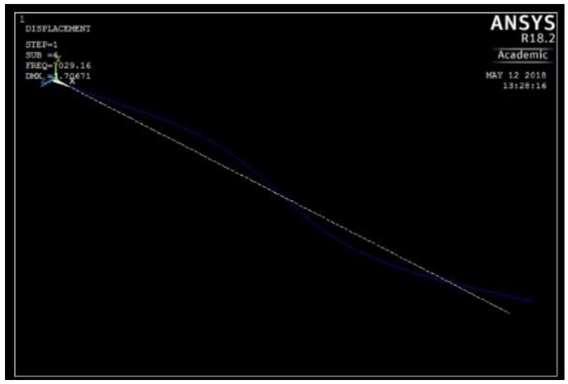
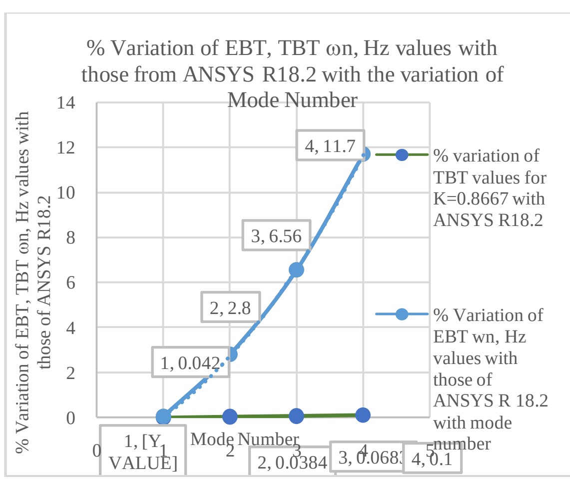


Fig 11. Principal Mode 4

Table 3. Comparison of AnsysR18.2 Results with the Results of EBT and TBT for Mild Steel Cantilever Beam of Length, L=137.5 mm:

Mode No.	ANSYS R18.2 ω_n, Hz	EBT ω_n, Hz	TBT ω_n, Hz K=0.8667	% variation of EBT values with ANSYS R18.2	% variation of TBT values for K=0.8667 with ANSYS R18.2
1	425.17	426.95	425.18	0.042	0.00235
2	2601.5	2675.55	2602.5	2.8	0.0384
3	7029.2	7.49E+03	7034	6.56	0.0683
4	13143	14681	13,156.70	11.7	0.1

From Table 3, we can observe that the percentage variation of EBT values with ANSYS R18.2 values for the natural frequency of the first four modes for the rectangular cantilever beam of length 137.5 mm is from 0.042% to 11.7%. And also the percentage variation of TBT values with ANSYS R18.2 values for the natural frequencies of the first four modes of a cantilever beam of length 137.5 mm and identical rectangular cross-section is from 0.00235% to 0.1% for the shear coefficient value, K=0.8667. The percentage variation of TBT values with ANSYS R18.2 values for the natural frequencies for the first four modes of a cantilever beam of length 137.5 mm and identical cross-section is from -0.47% to 0.17% for the shear coefficient value, K=0.8667. The values obtained for TBT are nearer to the ANSYSR18.2 values than those for EBT values for short cantilever beam. The same can be observed in the following Graph 2



Graph 2. Percentage variation of EBT, TBT Frequency, $\omega_{n,}$Hz values with those from ANSYS R18.2 for the first four Modes

IV. CONCLUSIONS

1. We can observe that the fundamental natural frequency values of a cantilever beam will be increased as the length of the cantilever beam is reduced keeping the cross-section constant. And also we can observe that the fundamental natural frequency values for different lengths of the identical rectangular cross-section cantilever beam from Timoshenko Beam Theory (TBT) are nearer to the ANSYS R18.2 values than those obtained from Euler-Bernoulli Theory (EBT).
2. The percentage variation of the fundamental natural frequencies for the decrease of the length of the cantilever beam obtained from the Euler-Bernoulli Theory (EBT) are more than those from the Timoshenko Beam Theory (TBT) when compared with the values obtained from ANSYS R18.2.
3. We can also observe that the variation of EBT and TBT values of fundamental natural frequency with those of ANSYS R18.2 for a fixed length of cantilever beam and identical cross-section is increased as the mode number is increased.
4. The fundamental natural frequencies obtained from Timoshenko Beam Theory (TBT) are nearer to the ANSYS R18.2 values than those values obtained from Euler-Bernoulli Theory (EBT).
5. Hence, we can say that for the analysis of free transverse vibrations of short cantilever beams either for lower or higher modes Timoshenko Beam Theory (TBT) is the best estimate than Euler-Bernoulli Beam Theory (EBT).

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